Hong Kong Mathematics Olympiad (2004 – 2005)
Heat Event (Group)
香港数学竞赛 (2004 – 2005)
初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 若 $x=\frac{19}{97}+\frac{19}{97}\times 2+\frac{19}{97}\times 3+\cdots+\frac{19}{97}\times 10$ 及 a 是最接近 x 的整数，求 a 的值。

Let $x=\frac{19}{97}+\frac{19}{97}\times 2+\frac{19}{97}\times 3+\cdots+\frac{19}{97}\times 10$ and a is the integer that is the closest to x, find the value of a.

2. 已知正方形 $ABCD$ 的面积是 130 cm^2 及圆 O 经过点 A、B、C 及 D。若圆 O 的面积是 $b\text{ cm}^2$，求 b 的值。(取 $\pi=3.14$)

Given that the area of a square $ABCD$ is equal to 130 cm^2 and a circle O passes through the points A, B, C and D. If the area of the circle O is $b\text{ cm}^2$, find the value of b.(take $\pi=3.14$)

3. 已知 p、q 和 r 是方程 $x^3-x^2+x-2=0$ 的三个不同的根。若 $Q=p^3+q^3+r^3$，求 Q 的值。

Given that p, q and r are distinct roots of the equation $x^3-x^2+x-2=0$. If $Q=p^3+q^3+r^3$, find the value of Q.

4. 当一个三位数减去它的各个数字的数字的和，其差还是一个三位数 $\overline{46x}$，求 x 的值。

When a 3-digit number minus the sum of the values of the three digits, the difference is a 3-digit number $\overline{46x}$, find the value of x.

5. 若 B 是整数且 $B>\left(\sqrt{2}+\sqrt{3}\right)^6$，求 B 最小可能的值。

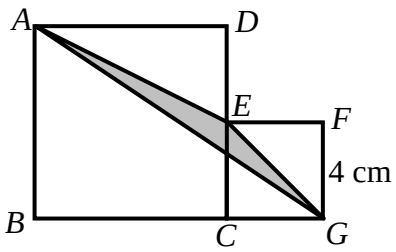
If B is an integer and $B>\left(\sqrt{2}+\sqrt{3}\right)^6$, find the smallest possible value of B.

6. 若一正八面体的边长为 1 cm，其体积为 f cm^3，求 f 的值。

Suppose the side of a regular octahedron is equal to 1 cm and the volume is equal to f cm^3, find the value of f.

7. 如图一，$ABCD$ 和 $CEFG$ 是两个正方形，$FG=4$ cm。若 ΔAEG 的面积是 g cm^2，求 g 的值。

In Figure 1, $ABCD$ and $CEFG$ are two squares and $FG=4$ cm. If the area of ΔAEG is equal to g cm^2, find the value of g.



图一

Figure 1

8. 设 x 为实数。若 h 是 x 的最大值使得 $2(\log_{\frac{1}{2}}x)^2+9\log_{\frac{1}{2}}x+9\le 0$，求 h 的值。

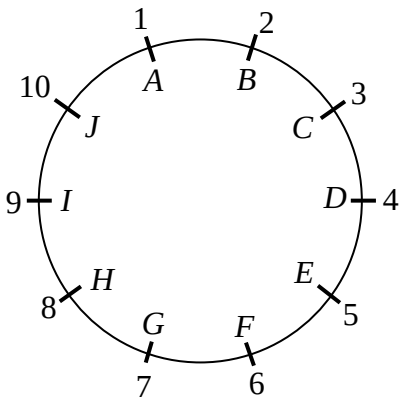
Let x be a real number. If h is the greatest value of x such that $2(\log_{\frac{1}{2}}x)^2+9\log_{\frac{1}{2}}x+9\le 0$, find the value of h.

9. 已知在三角形 ABC 内的一点 O 到三角形三边的垂线的长度均为 2 cm，而 ΔABC 的周界为 21 cm。若 ΔABC 的面积是 k cm^2，求 k 的值。

Given that the perpendicular distances from the point O to three sides of a triangle ABC are all equal to 2 cm and the perimeter of ΔABC is equal to 21 cm. If the area of ΔABC is equal to k cm^2, find the value of k.

10. 如图二，十人围成一圈，并依座号 $1, 2, 3, \ldots, 10$ 而坐。每人选择一个整数，分别是 $A, B, C, \ldots, J$，并将这个数字告诉他左右两个邻座的人。每人跟着算出他左右两个邻座所选的数的算术平均数。若各人所算出的平均数与其座号相等，求 F 的值。

In Figure 2, ten people are sitting in a round table with sitting numbers $1, 2, 3, \ldots, 10$ respectively. Each of them chooses an integer $A, B, C, \ldots, J$ respectively and tells the people on his left and right about his chosen number. Then each of them calculates the average number of the chosen numbers of his two neighborhoods and announces this average number. If all the announced average numbers are the same as the corresponding sitting numbers, find the value of F.



图二

Figure 2